A Unique Normal Form for Synonyms in the Propositional Calculus

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1. The Language L_1.

Let L_1 be a language for the propositional calculus defined as follows:

Primitive symbols: . –) (
$P_1, P_2, P_3, \ldots$ (propositional variables)

Rules of Formation: A_1 is a wff iff A_1 is a propositional variable, or A_2 and A_3 are wffs and A_1 is $\ulcorner -A_2 \urcorner$ or $\ulcorner (A_2 . A_3) \urcorner$

Abbreviations (Definitions): D1. $\ulcorner (A_1 \supset A_2) \urcorner$ for $\ulcorner -(A_1 . -A_2) \urcorner$

D2. $\ulcorner (A_1 \vee A_2) \urcorner$ for $\ulcorner -(-A_1 . -A_2) \urcorner$

D3. $\ulcorner (A_1 \equiv A_2) \urcorner$ for $\ulcorner [(A_1 \supset A_2) . (A_2 \supset A_1)] \urcorner$

It is to be assumed that the primitive and defined constants may be given the usual truth-functional interpretations.

2. Synonymity.

The following metalogical definition of 'Syn' is intended to provide a plausible partial formal correlate of the intuitive notion of being synonymous with, or meaning the same as:

*D1. 'A_1 SYN A_n' for 'There is some sequence, $\ulcorner (A_1, A_2 \ldots, A_{n-1}, A_n) \urcorner$ such that for each $\ulcorner A_i, A_{i+1} \urcorner$, A is just like A_{i+1} except that some wff A_j in A_i is replaced in A_{i+1} by A_k, and either

(i) A_j, A_k are definiens and definiendum, or vice versa, by D1 to D3, or

(ii) A_j is B and A_k is $\ulcorner -- B \urcorner$ or vice versa (DM), or

(iii) A_j is B and A_k is $\ulcorner (B.B) \urcorner$ or vice versa (Idem), or

(iv) A_j is $\ulcorner (B.C) \urcorner$ and A_k is $\ulcorner (C.B) \urcorner$ (Comm), or

(v) A_j is $\ulcorner [B.(C.D)] \urcorner$ and A_k is $\ulcorner [C.(B.D)] \urcorner$ (Assoc), or

(vi) A_j is $\ulcorner -[B.-(C.D)] \urcorner$ and A_k is $\ulcorner [-(B.-C).-(B.-D)] \urcorner$ or vice versa (Distr.) .'

Speaking more loosely, synonymity of propositional schemata is preserved by substitutions based only on interchange of definiens and definiendum, or principles of double negation, idempotence, commutation, association, and distribution.

The following theorems establish various properties of SYN, and some of its relations to logical equivalence:

<u>Theorem I.</u> <u>If A_1 SYN A_n then A_1 Eq A_n.</u>

<u>Proof</u>: By 'A_1 Eq A_n' ("A_1 is logically equivalent to A_n") we mean that $\ulcorner(A_1 \equiv A_n)\urcorner$ is a tautology, or a logical truth. Inspection of each of the rules (i) to (vi) shows that the pairs A_j, A_k in each rule are such that $\ulcorner(A_j \equiv A_k)\urcorner$ will be a tautology. Since each A_i, A_{i+1} differ only in the replacement of a component by a logically equivalent expression, it follows by the substitutivity of equivalents and the transitivity of logical equivalence in standard logic that A_1 will always be logically equivalent to A_n, if A_1 SYN A_n.

We have ignored, here, "logical equivalence" of alphabetic variants; as we ignored the synonymity of alphabetic variants in *DI. Relations between alphabetic variants could be included in both places, but at the expense of simplicity. We have chosen to ignore alphabetic variants, on the ground that they reflect merely notational conveniences; it is perfectly possible to set up a satisfactory logical language such that wffs must all have a certain alphabetic order and all alphabetic "variants" from this are not wffs. Though awkward transformations must be introduced, this shows the dispensability of alphabetic variants in the present discussion.

We will see below that the converse of Theorem I does not hold on at least three different independent grounds. There are a great many important classes of cases where A_1 is Eq to A_2, but not A_1 SYN A_2. According to Theorem 2 '(p ⊃ p)' and '(q ⊃ q)' are logically equivalent but not synonymous. According to Theorem 3 '(p.-p.q)' and '(p.-q.q)' are logically equivalent but not synonymous. and according to Theorem 8, '(p≡q)' and '(p.q) V (-p.-q)' are logically equivalent but not synonymous.

Theorem 2. If A_1 SYN A_n then A_n contains all and only the same variables as A_1.

Proof: Inspection of each of the rules (i) through (vi) shows that in each step A_i, A_{i+1}, the substituent and the substituens have the same components - only logical constants are changed. While logical groupings and constants may change, no non-logical primitive, no variable, can be added or entirely eliminated.

The converse of this, of course, does not hold; merely having the same variables is no guarantee of logical equivalence, much less of SYN.

Theorem 3. If A_1 SYN A_n then every variable, v_1, in A_1 occurs in the scope of an odd (even) number of negation signs (after reduction to primitive notation) if and only if v_1 occurs in the scope of an odd (even) number of negation signs in A_n (after reduction to primitive notation).

Proof: Inspection of Rules (ii) through (vi) shows that there is no change under (iii), (iv), or (v) in the number of negation signs within the scope of which any component will fall. In (ii) the changes always increase or decrease this number by two, thus preserve the oddness (evenness) of the number for any component, including variables. In (vi) the components B,C,D, will occur under an odd (even) number of negations signs in the substituent iff they occur under an odd (even) number of negations signs in the substituens.

Again, the converse of this, of course, does not hold; e.g., '(p.-q)' and '-(q.-p)' satisfy the consequent of Theorem 3, but are not logically equivalent and thus, by Theorem 1 and modus tollens, not SYN.

<u>Theorem 4. SYN is reflexive, symetrical, transitive.</u>

<u>Proof</u>: Always A_1 SYN A_1. For A_1 SYN $\ulcorner(A_1.A_1)\urcorner$ by *DI, (iii) and again $\ulcorner(A_1.A_1)\urcorner$SYN A_1 by *DI, (iii) - the 'vice versa' clause. Hence we get the sequence $\ulcorner A_1,(A_1.A_1),A_1\urcorner$ and thus, by *DI A_1SYN A_1. Thus SYN is reflexive.

Always if A_1SYN A_2 then A_2SYN A_1. For each of rules (i), (ii), (iii) and (vi) have a 'vice versa' clause,and rules (iv) and (v) don't need them since for any B, C, putting C for B and B for C will give the same result. Hence, since every step is symetrical, any sequence of steps $\ulcorner(A_1,\ldots,A_n)\urcorner$ if put in reverse order, $\ulcorner(A_n, A_{n-1},\ldots,A_1)\urcorner$ will satisfy *DI. Hence if A_1SYN A_n then A_nSYN A_1, i.e., SYN is symetrical.

Transitivity of SYN follows from *DI. Any sequence $\ulcorner(A_1,A_2,A_3)\urcorner$ which satisfies the conditions stated for A_1 SYN A_3, must also be such that the sequence $\ulcorner(A_1,A_2)\urcorner$ satisfies the conditions for A_1SYN A_2 and $\ulcorner(A_2,A_3)\urcorner$ satisfies the condition for A_2SYN A_3. Conversely, if for some A_1,A_2,A_3, the sequences $\ulcorner(A_1,A_2)\urcorner$ and $\ulcorner(A_2,A_3)\urcorner$ satisfy the conditions for A_1SYN A_2 and A_2SYN A_3 respectively, then we can form the sequence $\ulcorner(A_1,A_2,A_3)\urcorner$ in the assurance that it will satisfy the conditions for

A_1 SYN A_3. This argument can be extended for any A_1, A_n, A_{n+m} (n and m being finite.)

Let all abbreviations, be removed from some wff A, and for each variable v_1 let all these occurrences of v_1 in A which lie in the scope of an odd number of denial signs be underlined. Now let us treat underlined occurrences of each v_1 as occurrences of some new variable not occurring elsewhere in A, Let us call the underlined variables 'negative variables', the non-underlined variables 'positive variables', and the set of all negative variables plus the set of positive variables, the set of 'PN-variables'. Truth=tables which make assignments to PN-variables as if they were all distinct variables in the standard sense will be called 'PN-truth-tables'. Then,

Theorem 5. A_1 SYN A_n if and only if A_1 and A_n have exactly the same set of m PN-variables.[$k \leq m \leq 2k$, where k =(no. of variables)].

Proof: By Theorem 3 and the meaning of 'PN-variables' as given above. If k is the number of variables and m the number of PN-variables, $k \leq m \leq 2k$ since there can be, at most, k variables which occur both in the scope of odd number and in the scope of an even number of denial signs, and each variable can occur at most in two ways - under the scope of an odd, or an even, number of denial signs.

Theorem 6. If each PN-variable in A is assigned truth-values as if it were a distinct variable, then the PN-truth-table of A will be the same as the truth-table of A', where A' is the result of replacing in turn each underlined PN-variable in A by a variable not previously occurring in A.

Proof: By nature of truth-tables.

Theorem 7. If A_1' and A_n' are just like A_1 and A_n, respectively, except that each underlined variable in A_1 and A_n is replaced, in turn, at all its occurences, by a variable not occurring previously, then, if A_1 SYN A_n then A_1' SYN A_n'.

Proof: Let $A_1, \ldots, A_{n-1}, A_n$ be any sequence of wffs satisfying the definiens of *D1 (definition of SYN).

Then, by definition, A_1 SYN A_n. Now let A_1' be like A_1 except as described in the hypothesis. For each step, $\ulcorner A_i, A_{i+1} \urcorner$ in the sequence $\ulcorner A_1, \ldots, A_n, \urcorner$ there will now be a corresponding step $\ulcorner A_i', A_{i+1}', \urcorner$ in which the underlined variable occurences in A_i and A_{i+1} are replaced by the same new variables which replace them in A_1'. Every step goes through as before, by one of the rules (i) through (ii), none of the new variables being eliminated along the way, (by theorem 2), or occurring in the scope of an even number of denial signs in any step, (by theorem 3), so that the terminal step, ending with A_n' contains exactly the same substitutions for A_n as A_1' contained for A_1, and because the sequence $\ulcorner A_1', \ldots, A_n' \urcorner$ satisfies the definiens of *D1, with exactly the same justifications at each step as $\ulcorner A_1, \ldots, A_n \urcorner$, A_1' SYN A_n'.

<u>Theorem 8. If A_1 SYN A_n then the PN-truth-table of ⌜$(A_1 \equiv A_n)$⌝ is tautologous.</u>

<u>Proof</u>: By Theorem 6, the NP-truth-table for ⌜$(A_1 \equiv A_n)$⌝ is equivalent to the standard truth-table for ⌜$(A_1' \equiv A_n')$⌝ (Defining A_1' and A_n' in relation to A_1, A_n, as defined in Theorem 7). But also, by Theorem 7, if A_1 SYN A_n then A_1' SYN A_n'. Now suppose ⌜$(A_1' \equiv A_n')$⌝ is not tautologous; then, by Theorem 1 and modus tollens, A_1' is not SYN with A_n'. But in that case, by Theorem 7, and modus tollens, A_1 is not SYN with A_n. But this contradicts the hypothesis, A_1 SYN A_n. Hence, ⌜$(A_1' \equiv A_n')$⌝ must be tautologous, and Theorem 8 holds.

The converse of <u>Theorem 8</u> does not hold; e.g., the PN-truth-table of '(p ≡ (p.(pV-p))' is tautologous, but, by Theorem 3, 'p' is not SYN with '(p.(pV-p))'- though they are logically equivalent. Note also, the analogue of Theorem 8 for logical equivalence does not hold: '((p.q)V(-p.-q))' is logically equivalent to '((p⊃q).(q⊃p))' but the PN-truth-table of '((p.q)V(-p.-q)) ≡ ((p⊃q).(q⊃p))' is not tautologous.* (Put 'r' for negative occurrences of 'p' and 's' for negative occurrences of 'q'; then assign p=F, q=F, r=T, s=T).

3. <u>Normal Forms for Synonyms (NFS's).</u>

Now we wish to define a certain kind of normal form, which we will call a normal form for synonyms and abbreviate as 'NFS'. We also wish to prove (Theorem 9) that every wff is synonymous with some NFS, and define the relation, "an NFS <u>of</u> y is x" or "x is an NFS <u>of</u> y". Following this we shall prove that for every wff there is only one NFS which is synonymous with it; i.e., for every wff there is a unique normal form for its synonyms.

* Note: '(p ≡ (p. pV-q))' = A' if '(p ≡ (p.(pV-p))' = A.
'((p.q) V(-r.-s))' = A' if '((p.q) V (-p.-q)' = A
'(-(r.-q). -(s.-p))' = A' if '((p⊃q).(q⊃p))' = A

We define 'B_1 is an NFS' as follows

*D2. 'B_1 is an NFS' for 'B_1 has the form ⌜$(C_1.(C_2.(\ldots(C_n)))$⌝ and

(i) each C_i is either an elementary wff, E_j, (i.e., a variable or a negated variable), or a disjunction, D_j, of elementary wffs of the form ⌜$-(E_1.(E_2.(\ldots(E_m)))$⌝;

(ii) if C_i does not contain some PN-variable, v_1, which occurs elsewhere in B_1, then there is a D_j in B_1 which contains all the PN-variables in C_i plus v_1.

(iii) The components of B_1 are arranged according to a univocal alphabetic ordering, without repetitions, ⌜$(..(A_1.(A_1\ldots))..$⌝.

The effect of condition (ii) is that the normal forms for synonyms of A_1 are such that if they contain any 1st level conjunct C_i (i.e., a conjunct which does not lie in the scope of a negation outside itself) which contains a set, S, of PN-variables, then every larger disjunction of elementary wffs which contains the same set S, plus one or more other PN-variables appearing in A_1, will also be a first-order conjunct of any NFS of A_1. For example, if A_1 contains p,p,q only, and A_1SYN '(-p.-(-p.-q))', then the latter expression is not in normal form, by condition (ii), although it is by condition (i). To satisfy condition (ii), '(-p.-(-p.-q))' must be expanded to '(-p.(-(-p.p).(-(-q.p).(-(-p.-q).-(-p.(p.-q))))))', the three new disjunctions added being those which contain only PN-variables of the initial wff, but also all PN-variables which occur in smaller conjuncts found in the original wff.

We next prove:

<u>Theorem 9</u>. <u>For every wff A_1, there is some wff B_1 such that A_1SYN B_1 and B_i is an NFS.</u>

<u>Proof</u>: Given any A_1, it is easily seen that condition (i) can be satisfied; for it is well known that we can reduce any wff A_1 to conjunctive normal form in the sense of Hilbert and Ackermann, Ch. I, §3, using only double negation, idempotence, commutativity, association and distribution and DeMorgan's theorems. Rules (ii) through (vi) in *DI are equivalent to the first five principles, and by the definitions D1-D3 and double negation DeMorgan's laws follow; association and commutation will put the result in the required groupings.

To show that condition (ii) can always be satisfied for any A_1, let us suppose that condition (i) has been met, so that A_1 SYN A_2, where A_2 satisfies condition (i), i.e., has the form ⌜$(C_1.(C_2...(C_i.(...(C_n)))))$⌝. But suppose C_i lacks some PN-variable, v_1, which occurs elsewhere in A_2. If v_1 occurs elsewhere, it must occur in some conjunct C_j. By association and commutation it is possible to get an A_3, A_3 SYN A_2, such that ⌜$(C_i.C_j)$⌝ is a conjunct. Now C_j is either an elementary formula E_1 whose variable is v_1, or it is a disjunction of elementary wffs, one of which, E_1, contains v_1.

<u>Case: I</u> $A_3 = (\ldots((C_i.E_i)\ldots))$

(1) $(\ldots((C_i.E_i)\ldots))$

(2) $(\ldots(-(-(C_i.-(-E_i)))\ldots))$ (1),*DI(ii)DN(twice)

(3) $(\ldots(-(-(C_i.-(-E_i.-E_i)))\ldots))$ (2),*DI(iii)Idem.

(4) $(\ldots(-(-(C_i.--E_i).-(C_i.--E_i)))\ldots))$ (3),*DI(vi)Dist.

(5) $(\ldots(-(-(C_i.E_i).-(C_i.E_i)))\ldots))$ (4),*DI(ii)DN(twice)

(6) $(\ldots(-(-(C_i.E_i).-C_i).-(-(C_i.E_i).-E_i))\ldots))$ (5),*DI(vi)Dist.

(7) $(\ldots(-(-C_i.-(C_i.E_i)).-(-E_i.-(C_i.E_i)))\ldots))$ (6),*DI(iv)Comm.(twice)

(8) $(\ldots((-(-C_i.-C_i).-(-C_i.-E_i)).(-(-E_i.-C_i).-(-E_i.-E_i)))\ldots))$

(7),*DI(vi)Dist.(twice)

(9) $(\ldots(((-(-C_i).-(-C_i.-E_i)).(-(-E_i.-C_i).-(-E_i)))\ldots))$

(8),*DI(iii)Idem(twice)

(10) $(\ldots((C_i.-(-C_i.-E_i)).(-(-E_i.-C_i).E_i))\ldots))$ (9),*DI(ii)DN(twice)

(11) $(\ldots((C_i.(E_i.-(-C_i.-E_i)))\ldots))$ (10),*DI(iv),(v) and (iii) Assoc, Comm & Idem.

If v_i is negative, the E_i is initially a denied variable, and the '$-E_i$' in (11) will give way, by DN, to an v_i; otherwise E_i $\equiv \ulcorner\sim v_i\urcorner$ [redacted]. If C_i is an elementary wff, then either some $v_i = C_i$, or C_i is $\ulcorner -v_i \urcorner$, in which case $\ulcorner -C_i \urcorner$ in (11) is replaced through double negation by $\ulcorner v_i \urcorner$. If C_i is a disjunction, $\ulcorner -(E_2.E_3.\ldots.E_m) \urcorner$, then by double negation, $\ulcorner -(-C_i.-E_i) \urcorner$ becomes simply $\ulcorner -((E_2.E_3.\ldots.E_m).-E_i) \urcorner$, i.e., another disjunction of elementary wffs. The proper NFS grouping can then be restored by Association and Commutation (*DI, (iv) and (v)0. Thus in <u>case I</u>, condition (ii) for NFS is satisfied.

<u>Case 2</u>. A_3 =(...((C_i.D_j)...)) where D_j is a disjunction of elementary wffs including one, E_i, where the variable in E_i does not occur in C_i. Now we suppose that by association and commutation inside D_j, we can get a D_j', such that D_j' SYN D_j, and D_j'= -(E_i.A_4). Thus,

(1) (...((C_i.D_j)...))

(2) (...((C_i.-(E_i.A_4))...) (1), and Hyp. above

(3) (...(-(-(C_i.-(E_i.A_4))...) (2),*DI(ii)DN

(4) (...(-(-(C_i.-E_i).-(C_i.-A_4))...)) (3),*DI(vi)Dist.

(5) (...((-(-(C_i.-E_i).-C_i).-(-(C_i.-E_i).--A_4))...)) (4),*DI(vi)Dist

(6) (...((-(-C_i.-(C_i.-E_i)).-(A_4.-(C_i.-E_i)))...)) (5),*DI(iv) [illegible]

(7) (...((-(-C_i.-C_i).-(C_i.--E_i)).(-(A_4.-C_i).-(A_4.--E_i)...)) (6),*DI(vi)Dist [illegible]

(8) (...(C_i.(-(E_i.A_4).(-(E_i.-C_i).(-(A_4.-C_i)....))))) (7),*DI(ii),(iii) [illegible]

After removing double negations, in case C_i is a negated wff, at occurrences of ⌜-C_i⌝, we have condition (ii) of *D2 satisfied with respect to Case 2 for C_i and v_i.

By similar procedures condition (ii) in *D2 may be satisfied for every PN-variable, so that if a PN-variable occurs in any conjunct E_i or D_i, every D_j which contains all the PN-variables in E_i or D_i but no variable not occurring in A_i, will occur as a conjunct of B_i.

The satisfiability of condition (iii) in *D2, is left to the reader. Given a univocal alphabetical ordering, commutation and association (*DI(iv) and *DI(vi)) are sufficient to satisfy it; and when it is satisfied, repetitions ⌜...(A_i. [illegible] can be removed by association (*DI(v)0, commutation (*DI(iv)) and idempotence (*DI(iii)).

We now define 'B_1 is an NFS of A_1', or 'NFS(A_1,B_1)' as follows:

*D3 'NFS(A_1,B_1)' <u>for</u> 'A_1 is a wff and B_1 is an NFS and A_1SYN B_1'

And we wish to prove that for each A_1 there is at most one B_1,NFS(A_1,B_1).

For this we prove first the following theorem.

<u>Theorem 10</u>. <u>For any B_1 and B_2, if B_1 is an NFS and B_2 is an NFS and B_1SYN B_2, then $B_1 = B_2$.</u>

<u>Proof</u>: <u>Hypothesis 1</u>: B_1 is an NFS

<u>Hypothesis 2</u>: B_2 is an NFS

<u>Hypothesis 3</u>: B_1 SYN B_2

We show first that the class of components in B_1 has exactly the same members as the class of components in B_2; then the identity of the two can easily be established since the definition of NFS requires that these components be in one and only order and that repetitions be eliminated. We need only prove, then, that they must contain the same components.

By hypothesis 3, B_1SYN B_2. Hence, by Theorem 2, B_1 and B_2 contain all and only the same propositional variables, and by Theorem 5 all and only the same PN-variables. Further, by *D2,(ii) B_1 and B_2 each contain a conjunct C_o which is a disjunction of the set of all its PN-variables since if any disjunction,D_i, lacks a PN-variable which occurs elsewhere in B_1, then B_1 contains another disjunction, D_j, which contains all the PN-variables of D_i, plus the one it misses; hence some C_o must contain all PN-variables of B_1. <u>Now suppose that there is some conjunct C_i which is in B_1 but not in B_2.</u>

Since B_1 and B_2 are both NFS wffs, by *D2 (ii), B_1 will contain all disjunctions which contain all elementary wffs contained in C_i and one or more of the others in C_o; but also, B_2, which does not contain C_i, will not contain any disjunct which contains a sub-set of the elementary wffs in C_i and no more. Since B_1 and B_2,(being NFS-wffs) are conjunctions of denials of elementary wffs, and by the nature of truth-tables, the PN-truth-table for B_1 will be F and the PN-truth-table of B_2 will be T if each positive variable in C_i is F and each negative variable in C_i is T, while all other variables are T if positive and F if negative. But then, the PN-truth-table of $\ulcorner(B_1 \equiv B_2)\urcorner$ is not tautologous; thus, by Theorem 8, B_1 is not SYN with B_2. But this contradicts hypothesis 3. Hence, there can not be some conjunct in B_1 which is not in B_2; or by similar reasoning, vice versa. Hence B_1 and B_2 have all the same conjuncts.

Since to be NFS wffs they must be in the same order and free of repetitions; and since they have all the same components, they are identical.

Hence if (1),(2) and (3) then $B_1 = B_2$. Q.E.D.

Our uniqueness theorem for normal forms for synonyms now follows:

<u>Theorem 11. For every wff A_1, there is one and only wff, B_1, such that B_1 is the NFS of A_1.</u>

<u>Proof</u>: For every wff A_1, there is at least one wff B_1, which is a NFS of A_1 (Theorem 9). If there is some B_2 such that B_2 is an NFS of A_1, then A_1 SYN B_2 and B_1 SYN A_1, by *D2. But then by the transitivity of SYN (Theorem 3), B_1 SYN B_2. By hypothesis, B_1 is an NFS and B_2 is an NFS. But then, by Theorem 10, $B_1=B_2$. Hence,

$$(A_1)(\exists B_1)(NFS(A_1,B_1).(B_2)(NFS(A_1,B_2) \supset B_1=B_2))$$

4. <u>Comments</u>:

1. The relation"...is the NFS of..." is a Many-One relation, i.e., a function.

2. Hilbert and Ackermann present two sorts of normal forms. The first sort had two varieties, duals of each other, the conjunctive normal form and the disjunctive normal form. Hilbert and Ackermann asserted that these normal forms do not provide a unique normal form for each wff. The assertion was based on the assumption that meaning is extensional, or truth-functional, and thus that two expressions "have the same meaning" iff they are the same truth-functions, i.e., are logically equivalent. The example they gave, to show non-uniqueness, was'(p≡q)'. They defined '(p≡q)' as standing for "the sentence which is true iff p and q are both true or both false", i.e., for '((p.q)V(-p.-q))'; therefore, they said, '(p≡q)' 'means, therefore, that p and q have the same truth-value'. But the conjunctive normal form of'((p.q)V(-p.-q))' turns out to be'((pV-p).((pV-q).((qV-p).(qV-q))';' while the conjunctive normal form for '((-pVq).(-qVp))' which is also logically equivalent to '(p≡q)'(and hence in their terminology, has the same "meaning") is itself, and these two normal forms can not be reduced to, or gotten from each other. If the meaning of "sameness of meaning" is changed, as suggested in this paper, to

mean just those wffs which can be gotten from each other by the kinds of transformation allowed in reduction to conjunctive normal forms (or dually, for disjunctive normal forms) then, on this definition the two normal forms are not synonymous (do not have the same meaning) -- among other things one contains the tautologies '(-pVp)' and '(-qVq)' where the other does not. Along with the uniqueness of normal forms which follows by adopting this view of synonymity, comes a great variety of distinctions among wffs which are completely passed over if sameness of meaning is identified with logical equivalence. On the other hand, there is at hand a decision procedure for distinguishing wffs which are tautologous, inconsistent or neither without recourse to truth-tables or axiomatic systems; it is analogous to the test by reduction to conjunctive normal forms. So we have not, in the process, lost any distinctions of importance to logic.

A second sort of normal form, labeled by Hilbert and Ackermann "ausgezeichnete" (or "distinguished") normal forms, permitted a unique normal form for each wff, but with this drawback - there was no conjunctive normal form at all for tautologies and no disjunctive normal form at all for inconsistencies. These are the same as the "distributive normal forms" of Hintikka,so far they are found in the propositional calculus. The NFS normal forms above have the advantage of (1) being unique for every wff, tautologous, inconsistent, or neither, and (2) of being capable of distinguishing effectively between a wide variety of different kinds of tautologies, and inconsistencies, as well as a variety of different sorts of wffs, with the same truth-function.

Finally, the function, NFS(A_1,B_1) is an <u>intensional</u> function of functions, in the precise sense of <u>Principia Mathematica</u>, Vol.I, pp. 72-3. And by the same token SYN is an intensional relation and not an extensional relation, in the same sense. For the following may be true: NFS(A_1,B_1), A_1SYN B_1, and $\ulcorner(B_1 \equiv B_2)\urcorner$ may be tautologous, (making B_1 and B_2 coextensive), yet NFS(A_1,B_2) and A_1SYN B_2 may both be false, as in the case just discussed [let A_1 be (p≡q)', B_1 be the NFS of '((p.q)V(-p.-q))' and B_2 be the NFS of '((-pVq).(-qVp))']. But since our treatment has been rigorously syntactical, it can not be held that this view of "intensions" suffers from the kinds of mentalism, vagueness, or mystical opacity, which have so often been objectionable in other "intensional" logics.